On maximal averages along hypersurfaces

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The study of L^p-estimates for maximal averages M_S associated to isotropic dilates of a given, smooth hypersurface S in Euclidean space originated from E.M. Stein's seminal work on dimension free estimates for the Hardy-Littlewood maximal operator, in which he had studied the spherical maximal function.

By localization, one can reduce to studying small surface-patches S near a given point x^0. Denoting by p_c the minimal Lebesgue exponent such that M_S is L^p - bounded for $p > p_c$, I shall first explain a new "geometric" conjecture on how the critical exponent p_c might be determined by means of a geometric measure theoretic condition, which measures in some sense the order of contact of arbitrary ellipsoids with S.

The main part of the talk will then focus on hypersurfaces in $\mathbb{R}^3$, for which we are able by now to identify p_c for almost all analytic surfaces (with the exception of a small subclass of surfaces exhibiting singularities of type A according to Arnold's classification), by means of quantities which can be determined from associated Newton polyhedra. Besides the well-known notion of height, a new quantity, which we call the effective multiplicity, turns out to play a crucial role here.

Our recent results lead in particular to a proof of our "geometric" conjecture for all analytic 2-surfaces which are not of exceptional class, as well as the proof of a conjecture by Iosevich-Sawyer-Seeger for arbitrary analytic 2-surfaces.